Correct solution. Let the highest common divisor of x and $x+a$ be m. Then evidently the solution is

$$x=mr^2\ldots(1),$$
$$x+a=ms^2\ldots(2),$$
$$y=mrs\ldots(3).$$

From (1) and (2), $a=m(s^2-r^2)$.

Then if a is given, we can put s^2-r^2 equal to the different factors of a, and m equal to the remaining factor. s^2-r^2 may be put equal to any factor of a excepting the double of an odd number.

Example. $a=15$. If $a^2-r^2=15$, $s+r=15$ or 5, $s-r=1$ or 3,

$$\left.\begin{matrix} s=8 \\ r=7 \end{matrix}\right\} \text{ or } \left.\begin{matrix} s=4 \\ r=1 \end{matrix}\right\}, \; m=1.$$

These give $x=49$, $y=56$, $x=1$, $y=4$.

This last solution is omitted in Professor Griffin's table owing to his incorrect analysis.

If $s^2-r^2=5$, $m=3$, $x=12$, $y=18$.

If $s^2-r^2=3$, $m=5$, $x=5$, $y=10$.

179. Proposed by V. M. SPUNAR, Chicago, Ill.

Solve the equation in integers, $x^n+y^n+z^n+xyz=100x+10y+z$.

Solution by A. H. HOLMES, Brunswick, Maine.

Put $xyz=100x$. $\therefore yz=100$, and $z=100/y$.

$\therefore x^n+y^n+\dfrac{100^n}{y^n}=10y+\dfrac{100}{y}$; or $x^n=10y+\dfrac{100}{y}-y^n-\dfrac{100^n}{y^n}$.

For n even, $10y+\dfrac{100}{y}>y^n+\dfrac{100^n}{y^n}$.

Put $n=2$. Then $10y+\dfrac{100}{y}>y^2+\dfrac{10000}{y^2}$.

But $y=\dfrac{100}{z}=1, 2, 4, 5, 10, 20, 25, 50$, or 100. $\therefore n=1$ and $x=9y$.

$$\therefore n=1: \left|\begin{array}{l} x=9,\ 18,\ 36,\ 45,\ 90,\ 180,\ 225,\ 450,\ 900. \\ y=1,\ 2,\ 4,\ 5,\ 10,\ 20,\ 25,\ 50,\ 100. \\ z=100,\ 50,\ 25,\ 20,\ 10,\ 5,\ 4,\ 2,\ 1. \end{array}\right.$$

Put $xyz=10y$. $\therefore xz=10$, $z=\dfrac{10}{x}$ and $x^n+y^n+\dfrac{10^n}{x^n}=100x+\dfrac{10}{x}$ or $y^n=$ $100x+\dfrac{10}{x}-x^n-\dfrac{10^n}{x^n}$.

Put $n=1$. Then $y=99x$. But $x=10/z=1, 2, 5, 10$.

$$\therefore n=1 : \left| \begin{array}{l} x=1,\ 2,\ 5,\ 10, \\ y=99,\ 198,\ 495,\ 990. \\ z=10,\ 5,\ 2,\ 1. \end{array} \right.$$

Put $n=2$. Then $y^2=100x+10/x-x^2-100/x^2$. $\therefore x=1$ and 10.

$$\therefore n=2 : \left| \begin{array}{l} x=1,\ 10. \\ y=3,\ 30. \\ z=10,\ 1. \end{array} \right.$$

In neither case, when $y=1, 2, 4, 5, 10, 20, 25, 50, 100$, and $x=1, 2, 5, 10$, can $n=3$.

180. Proposed by **A. H. HOLMES**, Brunswick, Maine.

Find integral values for x and y in the following: $96x-96y+21=\square$.

Solution by the PROPOSER.

If we solve the equation

$$(1)\quad (24y+z)^2=25(24x+z-1),$$

for z we find

$$z=\frac{25-48y\pm5\sqrt{(96x-96y+21)}}{2}.$$

Since the coefficient of z^2 in (1) is unity, if for a pair of values of x and y, $96x-96y+21$ is a square, the corresponding value of z must be a rational integer. But no set of integral rational values of x, y, z will satisfy (1), for if the value of z is odd, the left hand member of (1) is odd and the right hand member is even, and vice versa for z even. Hence there are no integral values for x and y such that the given expression is a square.

☞Problems and solutions for this department should be sent to Dr. Wahlin, Urbana, Ill.

PROBLEMS FOR SOLUTION.

ALGEBRA.

353. Proposed by **DANIEL KRETH**, Oxford, Iowa.

Divide 2940 into two such factors that the square of one factor minus 21 will equal three times the other factor.

354. Proposed by **THEODORE L. DeLAND**, Treasury Department, Washington, D. C.

To find x in the following equation:

$$0.002\{6x-20[(1.05)^x-1]\}=0.012\{21[(1.05)^{x-1}-1]-(x-1)\}.$$